**Black Hole Thermodynamics – Maths**

Schwarzschild Radius:

This derivation is complex and arises from a solution to Einstein’s field equations for a non-rotating uncharged black hole.

Schwarzschild Density:

Temperature:

Hawking Radiation:

Case of Stefan’s Law (the total electromagnetic radiation energy emitted per unit time by a black body is given by ) with a sphere: .

Thermal Equilibrium:

* Find the rate of mass loss due to Hawking radiation.
* Find the rate of mass gain due to CBR.
* Equate them at equilibrium and solve for time.

By definition:

We know:

Therefore:

The temperature of CBR in terms of time is given by:

The black hole absorbs CBR:

Flux also is equivalent to:

Multiplying by gives:

In another formulation:

Substitute CBR:

Simplifying:

We know (the radiation constant):

So:

Therefore:

Equating the two expressions:

Substitute area and Hawking radiation:

Substitute temperature:

Square root: